CAMS/00-08
UM-TH-00-25
hep-th/0010025

Multi-centered black holes in gauged $D = 5$ supergravity

James T. Liu [†], W. A. Sabra [‡]

[†] *Randall Laboratory, Department of Physics, University of Michigan, Ann Arbor, MI 48109–1120*

[‡] *Center for Advanced Mathematical Sciences (CAMS) and Physics Department, American University of Beirut, Lebanon*

ABSTRACT

One of the important consequences of the no-force condition for BPS states is the existence of stable static multi-center solutions, at least in ungauged supergravities. This observation has been at the heart of many developments in brane physics, including the construction of intersecting branes and reduced symmetry D-brane configurations corresponding to the Coulomb branch of the gauge theory. However the search for multi-center solutions to gauged supergravities has proven rather elusive. Because of the background curvature, it appears such solutions cannot be static. Nevertheless even allowing for time dependence, general multi-center solutions to gauged supergravity have yet to be constructed. In this letter we investigate the construction of such solutions for the case of $D = 5$, $N = 2$ gauged supergravity coupled to an arbitrary number of vector multiplets. Formally, we find a family of time dependent multi-center black hole solutions which are easily generalized to the case of AdS supergravities in general dimensions. While these are not true solutions, as they have a complex metric and gauge potential, they may be related to a Wick rotated theory or to a theory where the coupling is taken to be imaginary. These solutions thus provide a partial realization of true multi-center black-holes in gauged supergravities.

1 Introduction

There has been much interest in the study of black hole solutions of ungauged and gauged supergravity theory in recent years. This is to a large extent motivated by the study of duality symmetries and their relation to the non-perturbative limits of string theory and M theory. In addition to providing the foundation for a microscopic understanding of black hole physics, such explicit solutions play an important role in the conjectured AdS/CFT correspondence. Here, the anti-de Sitter geometry arises as the vacuum of gauged supergravities in various dimensions. Thus black holes in gauged supergravities are especially relevant for investigations of AdS/CFT, and the related brane-world models.

Consider, for example, the $\mathrm{AdS}_5 \times S^5$ compactification of type IIB theory, which gives $D = 5$, $N = 8$ gauged supergravity. The isometries of S^5 lead to a $SO(6)$ gauging, which in turn may be identified with the $SO(6)$ R-symmetry of the $D = 4$, $N = 4$ super-Yang-Mills theory on the boundary. The AdS/CFT duality then indicates that R-charged black holes would couple to corresponding R-charged operators of the $N = 4$ SYM theory. Such black holes, both supersymmetric [1] and non-supersymmetric (non-extremal) [2], have recently been constructed in gauged $N = 2$ supergravity, which is a consistent truncation of the $N = 8$ theory. While the abelian truncation of $N = 8$ supergravity yields exactly three vector multiplets (corresponding to the STU model), the solutions were in fact obtained for the $N = 2$ theory coupled to an arbitrary number of vector multiplets.

Since the supersymmetric R-charged black holes are expected to satisfy a BPS-type no-force condition, one may believe that extremal multi-center solutions would also exist. However, unlike for their ungauged counterparts, where multi-center black hole and p-brane configurations are well explored and understood, only few results have been obtained for multi-center solutions in gauged supergravities. The difficulty here lies in the fact that such R-charged black holes are asymptotic to anti-de Sitter space. The presence of the negative cosmological constant suggests that only single-center solutions will be static. While time-dependence is perhaps natural in a cosmological background, this added complication has yet to be fully considered in constructing black hole solutions to gauged supergravities.

Previously, multi-center black holes that are asymptotic to de Sitter space have been obtained in [3, 4]. As expected, these solutions are time-dependent.

Nevertheless the time dependence enters in a natural manner, and asymptotically yields the cosmological metric

$$ds^2 = -dt^2 + e^{-2gt} d\vec{x} \cdot d\vec{x}, \tag{1}$$

which has constant positive curvature. On the other hand, for the present purpose, one would actually like to obtain solutions in a background of negative curvature, since $D = 5$, $N = 2$ gauged supergravity naturally leads to an anti-de Sitter vacuum. While such multi-center solutions remain elusive, they may formally be obtained by sending $t \rightarrow it$ in the de Sitter metric, (1). Alternatively, this Wick rotation could be avoided by instead taking an imaginary coupling g [4]. Of course, neither of these possibilities are entirely satisfactory. But nevertheless this approach provides a partial solution to the problem of constructing multi-center anti-de Sitter black holes. Furthermore, as many investigations of the AdS/CFT conjecture have in fact been performed in Euclidean space, it is perhaps not unreasonable after all to focus on the Wick rotated theory (although in the present case, this Wick rotation yields a pure imaginary gauge potential).

The multi-center solutions of [3, 4] have been constructed in the context of a cosmological Einstein/Maxwell theory. As shown in [4], up to an imaginary coupling g, this theory may be viewed as the bosonic sector of pure $D = 5$, $N = 2$ supergravity. Within this framework, the black holes were shown to be supersymmetric through the explicit construction of Killing spinors [4]. In this paper, we extend the work of [3, 4] to the case of $N = 2$ supergravity with an arbitrary number of vector multiplets. We construct general supersymmetric multi-centered black hole solutions and their corresponding Killing spinors. Some of these $N = 2$ models are the gauged versions of those obtained from M-theory compactified on a Calabi-Yau threefold.

Note that in five dimensions, supersymmetric black holes for the pure $N = 2$ ungauged supergravity were first constructed in [5]. The metric for these black holes are of the Tanghelini form [6]. Subsequently, extreme black holes for $N = 2$ ungauged supergravity coupled to abelian vector multiplets were considered in [7, 8, 9]. An important feature of these black holes is that, for those with non-singular horizons, the entropy can be expressed in terms of the extremum of the central charge. Furthermore, the scalar fields take fixed values at the horizon independent of their initial values at spatial infinity [10]. BPS black holes were constructed in four-dimensional $N = 2$ gauged supergravity in [11].

2 $D = 5$, $N = 2$ Gauged Supergravity

We start with five-dimensional $N = 2$ gauged supergravity coupled to $n-1$ abelian vector multiplets [12, 13]. The fields in this theory consists of a graviton $g_{\mu\nu}$, gravitino ψ_μ, n vector potentials A^I_μ ($I = 1, 2, \ldots, n$), $n-1$ gauginos λ_i and $n-1$ scalars ϕ^i ($i = 1, 2, \ldots, n-1$). The bosonic part of the Lagrangian is given by[1]

$$\begin{aligned} e^{-1}\mathcal{L} &= \frac{1}{2}R + g^2 V - \frac{1}{4}G_{IJ}F^I_{\mu\nu}F^{\mu\nu J} - \frac{1}{2}\mathcal{G}_{ij}\partial_\mu\phi^i\partial^\mu\phi^j \\ &\quad + \frac{e^{-1}}{48}\epsilon^{\mu\nu\rho\sigma\lambda}C_{IJK}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda. \end{aligned} \tag{2}$$

The scalar potential V is given by

$$V(X) = V_I V_J \left(6X^I X^J - \frac{9}{2}\mathcal{G}^{ij}\partial_i X^I \partial_j X^J\right), \tag{3}$$

where $X^I \equiv X^I(\phi^i)$ represent the real scalar fields, and satisfy the condition $\mathcal{V} = 1$ where

$$\mathcal{V} = \frac{1}{6}C_{IJK}X^I X^J X^K. \tag{4}$$

This homogeneous cubic polynomial $\mathcal{V}$ defines a "very special geometry" of the $N = 2$ theory.

The physical quantities in (2) can all be expressed in terms of $\mathcal{V}$ according to

$$\begin{aligned} G_{IJ} &= -\frac{1}{2}\partial_I\partial_J \log\mathcal{V}\Big|_{\mathcal{V}=1}, \\ \mathcal{G}_{ij} &= \partial_i X^I \partial_j X^J G_{IJ}\Big|_{\mathcal{V}=1}, \end{aligned} \tag{5}$$

where ∂_i and ∂_I refer, respectively, to a partial derivative with respect to the scalar field ϕ^i and X^I. Furthermore, the constants V_I that arise in (3) specify the appropriate linear combination of the vectors that comprise the $N = 2$ graviphoton, $\mathcal{A}_\mu = V_I A^I_\mu$.

[1]Our conventions are as follows: We use the metric $\eta^{ab} = (-,+,+,+,+)$ and Clifford algebra $\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}$. The gravitationally covariant derivative on spinors is $\nabla_\mu = \partial_\mu + \frac{1}{4}\omega_{\mu ab}\Gamma^{ab}$ where $\omega_{\mu ab}$ is the spin connection. Finally, antisymmetrization is with weight one, so $\Gamma^{a_1 a_2 \cdots a_n} = \frac{1}{n!}\Gamma^{[a_1}\Gamma^{a_2}\cdots\Gamma^{a_n]}$.

For a Calabi-Yau compactification of M-theory, $\mathcal{V}$ denotes the intersection form, and X^I and $X_I \equiv \frac{1}{6}C_{IJK}X^JX^K$ correspond to the size of the two- and four-cycles of the Calabi-Yau threefold. Here C_{IJK} are the intersection numbers of the threefold. In this Calabi-Yau case, n is given by the Hodge number $h_{(1,1)}$ and C_{IJK} are the topological intersection numbers. Some useful relations from very special geometry are:

$$\begin{array}{c} X^IX_I = 1, \qquad \partial_i X_I = -\frac{2}{3}G_{IJ}\partial_i X^J, \qquad X_I = \frac{2}{3}G_{IJ}X^J, \\ X_I\partial_i X^I = X^I\partial_i X_I = 0. \end{array} \tag{6}$$

In gauged supergravity theories, the supersymmetry transformations get modified by g-dependent terms. For a bosonic background, the variations of the gravitino and gauginos are given by [1, 2]:

$$\begin{aligned} \delta\psi_\mu &= \left[\mathcal{D}_\mu + \frac{i}{8}X_I(\Gamma_\mu{}^{\nu\rho} - 4\delta_\mu^\nu\Gamma^\rho)F^I_{\nu\rho} + \frac{1}{2}g\Gamma_\mu X^I V_I\right]\epsilon, \\ \delta\lambda_i &= \left[-\frac{1}{4}G_{IJ}\Gamma^{\mu\nu}F^J_{\mu\nu} + \frac{3i}{4}\Gamma^\mu\partial_\mu X_I + \frac{3i}{2}gV_I\right]\partial_i X^I\epsilon. \end{aligned} \tag{7}$$

Here $\mathcal{D}_\mu$ is the fully gauge and gravitationally covariant derivative,

$$\mathcal{D}_\mu\epsilon = \left[\nabla_\mu - \frac{3i}{2}gV_IA^I_\mu\right]\epsilon. \tag{8}$$

The ungauged theory is obtained in the (smooth) limit $g \to 0$.

2.1 Multi-centered solutions in the ungauged theory

Before proceeding to the gauged theory, it is instructive to first review the corresponding black hole solutions in the ungauged theory [7, 8, 9, 14]. Using isotropic coordinates, it was found that, for the non-rotating cases, these solutions can be brought to the form

$$\begin{aligned} ds^2 &= -e^{-4U}dt^2 + e^{2U}d\vec{x}\cdot d\vec{x}, \\ A^I_t &= e^{-2U}X^I, \\ X_I &= \tfrac{1}{3}e^{-2U}H_I, \end{aligned} \tag{9}$$

where H_I are a set of harmonic functions,

$$H_I = h_I + \sum_{j=1}^{N}\frac{q_{Ij}}{|\vec{x}-\vec{x}_j|^2}. \tag{10}$$

It is convenient to define the rescaled coordinates

$$Y_I = e^{2U} X_I, \qquad Y^I = e^U X^I, \tag{11}$$

in which case the function U may be written as

$$e^{3U} = \frac{1}{6} C_{IJK} Y^I Y^J Y^K. \tag{12}$$

Note that the constants h_I are related to the values of the scalars at infinity, while the $q_{I\,j}$ are the electric charges.

It is straightforward to verify that this is a half BPS solution. In the absence of gauging, the supersymmetry transformations (7) reduce to

$$\begin{aligned}
\delta\psi_\mu &= \left[\nabla_\mu + \frac{i}{8} X_I ({\Gamma_\mu}^{\nu\rho} - 4\delta_\mu^\nu \Gamma^\rho) {F_{\nu\rho}}^I\right]\epsilon, \\
\delta\lambda_i &= \left[\frac{3}{8}\Gamma^{\mu\nu} F^I_{\mu\nu} \partial_i X_I - \frac{i}{2} g_{ij} \Gamma^\mu \partial_\mu \phi^j\right]\epsilon \\
&= \left[-\frac{1}{4} G_{IJ} \Gamma^{\mu\nu} F^J_{\mu\nu} + \frac{3i}{4} \Gamma^\mu \partial_\mu X_I\right] \partial_i X^I \epsilon.
\end{aligned} \tag{13}$$

As usual, the resulting Killing spinor equations are solved by balancing the gauge fields with the metric for $\delta\psi_\mu$, and the gauge fields with the scalars for $\delta\lambda_i$. The resulting Killing spinors are given by $\epsilon = e^{-U}\epsilon_0$ where ϵ_0 is a constant spinor satisfying $\Gamma_0\epsilon_0 = i\epsilon_0$.

In the absence of scalar fields, *i.e.* for pure $N = 2$ supergravity theory, this solution reduces to [5]

$$\begin{aligned}
ds^2 &= -H^{-2} dt^2 + H d\vec{x} \cdot d\vec{x}, \\
A_t &= 3H^{-1}.
\end{aligned} \tag{14}$$

For a single center black hole, the harmonic function may be written as $H = 1 + \frac{q}{r^2}$, so that

$$ds^2 = -\left(1 + \frac{q}{r^2}\right)^{-2} dt^2 + \left(1 + \frac{q}{r^2}\right)(dr^2 + r^2 d\Omega_3^2). \tag{15}$$

This metric may be rewritten in a Schwarzschild form by defining a new radial coordinate $\rho^2 = r^2 + q$, in which case the solution becomes

$$\begin{aligned}
ds^2 &= -\left(1 - \frac{q}{\rho^2}\right)^2 dt^2 + \left(1 - \frac{q}{\rho^2}\right)^{-2} d\rho^2 + \rho^2 d\Omega_3^2, \\
F_{t\rho} &= -3\partial_\rho\left(1 - \frac{q}{\rho^2}\right).
\end{aligned} \tag{16}$$

2.2 Single-centered solutions in the gauged theory

Turning on the gauging now introduces a scalar potential which has the effect of generating a negative cosmological constant. Spherically symmetric BPS electric solutions were found for this case in [1, 2] and are given by

$$\begin{aligned} ds^2 &= -e^{-4U} f dt^2 + e^{2U}(f^{-1}dr^2 + r^2 d\Omega_3^2), \\ A_t^I &= e^{-2U} X^I, \\ X_I &= \tfrac{1}{3} e^{-2U} H_I, \end{aligned} \tag{17}$$

where

$$f = 1 + g^2 r^2 e^{6U}. \tag{18}$$

These solutions are similar to those of the ungauged theory, (9), with the only modification being the introduction of the function f which depends on the square of the coupling constant g; for $g = 0$, we recover the solutions of the ungauged case. The form of this metric, with the function f, is identical to that used in "blackening" the standard p-brane solutions [15]. This was in fact used to advantage in [2, 16] to blacken these AdS solutions.

The solutions (17) admit Killing spinors satisfying the projection condition [1]

$$\epsilon = -f^{-1/2}\Big(i\Gamma_0 + gre^{3U}\Gamma_1\Big)\epsilon. \tag{19}$$

As a result, these black holes preserve half of the supersymmetry of the $N = 2$ theory. While for $g = 0$ this condition reduces to the corresponding one in the ungauged theory, for $g \neq 0$ it has a somewhat unusual form, at least from a p-brane point of view. More details and the solution of the Killing spinor equations, (7), are given in [1, 2] (see also [17]).

For black p-branes, the absence of a no-force condition precludes the existence of static multi-center solutions. Along these same lines, this spherically symmetric ansatz, and in particular the explicit use of the radial coordinate in (18), prevents the above solution from being generalized to the multi-center case. However, unlike for the case of black p-branes, here the solutions we seek are still BPS. Although such solutions would not be static due the cosmological background, supersymmetry ought to provide a useful guide in their construction.

3 Multi-centered solutions in the gauged theory

The difficulty in obtaining multi-center solutions within the above framework may be made evident by turning off the electric charges, whereupon the solution, (17), reduces to the AdS vacuum written in the form

$$ds^2 = -(1 + g^2 r^2)dt^2 + \frac{dr^2}{1 + g^2 r^2} + r^2 d\Omega_3^2. \tag{20}$$

This form of the AdS metric singles out a preferred radial coordinate, and hence is ill suited as a starting point for the construction of multi-centered solutions. Instead, it would be more natural to use an isotropic metric ansatz of the form

$$ds^2 = -e^{2A}dt^2 + e^{2B}d\vec{x} \cdot d\vec{x}, \tag{21}$$

where A and B are functions of $(t, \vec{x})$.

Before turning to the full construction of the solution, it is instructive to examine the form of the AdS vacuum given this isotropic ansatz. Unfortunately it is at this stage that one encounters the difficulty of confronting either imaginary time or coupling. For a multi-center solution, we insist on starting with a space-isotropic[2] AdS vacuum metric, so that A and B can only be functions of time. In this case, $A(t)$ may be transformed away, leaving only $B(t)$ to consider. The natural choice $B = -2gt$, given in (1), however yields a constant positive curvature metric

$$R_{\mu\nu\rho\sigma} = g^2(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}). \tag{22}$$

Thus one is left with little choice but to take $gt \rightarrow igt$, yielding

$$ds^2 = -dt^2 + e^{-2igt}d\vec{x} \cdot d\vec{x}. \tag{23}$$

To obtain a real metric, one may either performing a Wick rotation[3], $t \rightarrow it$, or transform to imaginary coupling $g \rightarrow ig$.

[2]If this condition were relaxed, one could select a 'preferred' space coordinate, z, and write the metric in Poincare form, $A = B = -\log gz$. However this would not lead to a suitable background from which to construct *localized* black holes.

[3]Note, however, that in Wick rotating the solution, the gauge potential A_t^I becomes pure imaginary. Thus in either case one ends up with imaginary or complex quantities in the gauge sector.

3.1 Construction of the solutions

With this AdS vacuum in mind, our aim is to find supersymmetric multi-black hole solutions through examination of the Killing spinor equations. While these first order equations are often more convenient to work with, the complete solution must still be fixed through the use of at least one equation of motion. The convenient choice here is to use the Bianchi identities and equations of motion of the gauge fields of the theory.

We start with the supersymmetry transformation of the gravitino. For a bosonic background and the time-dependent ansatz, this becomes

$$\begin{aligned}\delta\psi_t &= \left[\partial_t - \tfrac{3i}{2} g V_I A_t^I + \tfrac{1}{2} g e^A X^I V_I \Gamma_0\right]\epsilon - \tfrac{1}{2}\Gamma^m\left[\partial_m e^A \Gamma_0 + i X_I F_{tm}{}^I\right]\epsilon,\\ \delta\psi_m &= \left[\partial_m - \tfrac{i}{2} e^{-A} X_I F_{tm}{}^I \Gamma_0\right]\epsilon + \tfrac{1}{4}\Gamma_m{}^n\left[2\partial_n B + i e^{-A} X_I F_{tn}{}^I \Gamma_0\right]\epsilon\\ &\quad -\tfrac{1}{2}\Gamma_m\left[e^{-A}\partial_t B \Gamma_0 - g X^I V_I\right]\epsilon.\end{aligned} \tag{24}$$

Note that the corresponding transformation in the ungauged limit is restored by setting $g = 0$ and choosing all quantities to be time independent. The conditions for obtaining a vanishing transformation may then be grouped into ones common to the ungauged limit:

$$\begin{aligned}\left[e^A \partial_m A - i X_I F_{tm}{}^I \Gamma_0\right]\epsilon &= 0,\\ \left[e^A \partial_m(-2B) - i X_I F_{tm}{}^I \Gamma_0\right]\epsilon &= 0,\\ \left[2e^A \partial_m - i X_I F_{tm}{}^I \Gamma_0\right]\epsilon &= 0\end{aligned} \tag{25}$$

and new conditions specific to the gauged supergravity theory:

$$\begin{aligned}\left[2\partial_t - 3ig V_I A_t^I + g e^A X^I V_I \Gamma_0\right]\epsilon &= 0,\\ \left[\partial_t B + g e^A X^I V_I \Gamma_0\right]\epsilon &= 0.\end{aligned} \tag{26}$$

In parallel with the ungauged theory, the vanishing of the first set of equations, (25), suggests the imposition of the condition $\Gamma_0\epsilon = i\epsilon$ on the Killing spinors. This leads to the requirement

$$X_I F_{tm}{}^I = -\partial_m e^A, \tag{27}$$

as well as a relation between the metric functions A and B,

$$B(t,\vec{x}) = -\tfrac{1}{2} A(t,\vec{x}) + f(t). \tag{28}$$

In addition, the Killing spinor must satisfy

$$\epsilon(t,\vec{x}) = e^{\frac{1}{2}A(t,\vec{x})}\hat{\epsilon}(t). \tag{29}$$

We will complete the construction of the Killing spinors and return to the additional conditions, (26), below.

For now, we turn to the gaugino transformation, given by

$$\begin{aligned} \delta\lambda_i &= -\tfrac{3i}{4}\left[e^{-A}\partial_t X_I \Gamma_0 - 2gV_I\right]\partial_i X^I \epsilon \\ &\quad -\tfrac{1}{4}\Gamma^m\left[2e^{-A}G_{IJ}F_{tm}{}^J\Gamma_0 - 3i\partial_m X_I\right]\partial_i X^I\epsilon. \end{aligned} \tag{30}$$

Thus two equations are obtained:

$$\left[2e^{-A}G_{IJ}F^J_{tm} + 3i\partial_m X_I\Gamma_0\right]\partial_i X^I\epsilon = 0, \tag{31}$$

$$\left[\partial_t X_I + 2ge^A V_I \Gamma_0\right]\partial_i X^I\epsilon = 0. \tag{32}$$

The first equation, also present in the ungauged theory, together with (27) implies the following connection:

$$G_{IJ}F^J_{tm} = \frac{3}{2}e^{2A}\partial_m(e^{-A}X_I) \tag{33}$$

This can be verified by using the relations (6). In particular, combining (33) with (6), we obtain

$$\begin{aligned} e^{-A}G_{IJ}F_{tm}{}^J\partial_i X^I &= \tfrac{3}{2}e^A\partial_m(e^{-A}X_I)\partial_i X^I = \tfrac{3}{2}\partial_m X_I \partial_i X^I, \\ X_I F_{tm}{}^I = \tfrac{2}{3}G_{IJ}X^J F^I_{tm} &= e^{2A}\partial_m(e^{-A}X_J)X^J = -\partial_m e^A, \end{aligned} \tag{34}$$

which solves both (27) and (31). Therefore one can easily deduce that the gauge fields and scalars are related according to

$$A^I_t = e^A X^I. \tag{35}$$

To no surprise, this relation is identical to that found previously for the multi-center solution in the ungauged theory, (9), and for the single-center solution in the gauged theory, (17).

Until now we have only used supersymmetry to fix the relation between the bosonic fields of the solution. To proceed, we turn to the equation of motion for the gauge fields:

$$\nabla_\nu(G_{IJ}F^{\mu\nu\,J}) = \tfrac{1}{16}C_{IJK}\epsilon^{\mu\nu\lambda\rho\sigma}F^J_{\nu\lambda}F^K_{\rho\sigma}. \tag{36}$$

For a purely electric solution, the right hand side of the equation vanishes. There are two cases, corresponding to $\mu = t$ and $\mu = n$. Inserting the metric ansatz, (21), and making use of the relations (28) and (35), the gauge equation of motion then yields

$$\begin{aligned} \partial_m\partial_m(e^{-A+2f(t)}X_I) &= 0, \\ \partial_m\partial_t(e^{-A+2f(t)}X_I) &= 0. \end{aligned} \tag{37}$$

The first equation suggests a solution in terms of a harmonic function

$$e^{-A}X_I = \frac{1}{3}H_I(t,\vec{x}), \tag{38}$$

while the second equation specifies the time dependence of H_I. The factor of $1/3$ proves to be a convenient normalization [1]. As shown in [3, 4], the solution is given simply by choosing

$$H_I \equiv H_I(e^{f(t)}\vec{x}) = h_I + \sum_{j=1}^{N} \frac{q_{I\,j}}{e^{2f(t)}|\vec{x}-\vec{x}_j|^2}. \tag{39}$$

At this stage, we have satisfied all Killing spinor equations common to both the gauged and ungauged cases. What remains to be worked out are the new equations specific to the gauged theory, (26) for the gravitino and (32) for the gaugino. The function $f(t)$ can be obtained from the vanishing of the gaugino transformation, which gives

$$\left[\partial_t H_I + 6igV_I\right]\partial_i X^I \epsilon = 0, \tag{40}$$

where we have used the special geometry relation $X_I\partial_i X^I = 0$ to eliminate a term proportional to the time derivative of e^A. In fact, this same relation allows the above equation to be satisfied provided the quantity in the brackets is proportional to X_I and thus H_I. Since

$$\partial_t H_I = -2(H_I - h_I)\partial_t f, \tag{41}$$

we find the solution $f(t) = -igt$, where we have used $V_I = h_I/3$. This imaginary time solution yields a metric

$$ds^2 = -e^{2A}dt^2 + e^{-A}e^{-2igt}d\vec{x}\cdot d\vec{x}, \tag{42}$$

which is asymptotic to the vacuum AdS metric of (23). The metric function A may be determined from the underlying very special geometry, which implies that

$$e^{-\frac{3}{2}A} = \mathcal{V}(Y) = \frac{1}{6}C_{IJK}Y^IY^JY^K. \tag{43}$$

3.2 Killing spinors

We now return to the additional gravitino variations, (26), and solve for the Killing spinors. Using the solution for the gauge fields, (35), we obtain the following two equations for the Killing spinor:

$$\begin{aligned} \left[\partial_t(-B) - ige^A X^I V_I\right]\epsilon = 0, \\ \left[\partial_t - ige^A X^I V_I\right]\epsilon = 0. \end{aligned} \tag{44}$$

Combining these equations, we see that the Killing spinor must have the form

$$\epsilon(t,\vec{x}) = e^{-B(t,\vec{x})}\tilde{\epsilon}(\vec{x}) = e^{\frac{1}{2}A(t,\vec{x})}e^{-f(t)}\tilde{\epsilon}(\vec{x}), \tag{45}$$

which is only consistent with the previous condition, (29), for constant $\tilde{\epsilon}(\vec{x})$, *i.e.* $\tilde{\epsilon}(\vec{x}) = \epsilon_0$ and $\hat{\epsilon}(t) = e^{-f(t)}\epsilon_0$.

In order to check whether the first equation of (44) is satisfied we evaluate the quantity $gX^I V_I$. This can be done using our ansatz and the relations of very special geometry. First, using our solution, we write

$$\begin{aligned} gX^I V_I &= \frac{1}{3} g X^I (H_I - (H_I - h_I)) \\ &= ge^{-A} - \frac{1}{2}\frac{g}{\partial_t f}\partial_t e^{-A}, \end{aligned} \tag{46}$$

where we have used $e^{-A} = \frac{1}{3}X^I H_I$ as well as the relation (41) for the time derivative of H_I. From this, we obtain

$$gX^I V_I = \frac{g}{\partial_t f} e^{-A}\partial_t(f - \tfrac{1}{2}A) = \frac{g}{\partial_t f} e^{-A}\partial_t B, \tag{47}$$

which indeed satisfies (44) since $f(t) = -igt$. As a result, the Killing spinor has the simple expression

$$\epsilon = e^{-B}\epsilon_0 = e^{\frac{1}{2}A} e^{igt}\epsilon_0. \tag{48}$$

For $g = 0$, this expression for the Killing spinor, as well as the entire solution, reduces to that of the ungauged theory. Furthermore, the periodic time coordinate in AdS is apparent in (48), with $t \to t + 2\pi/g$.

4 Discussion

To summarize, we have explored the construction of multi-center black hole solutions to $D = 5$, $N = 2$ gauged supergravity. Extending the work of [3, 4], we have found imaginary time (or imaginary coupling) solutions breaking exactly half of the supersymmetry. These solutions may be written in the form

$$\begin{aligned} ds^2 &= -e^{2A}dt^2 + e^{-A}e^{-2igt}d\vec{x}\cdot d\vec{x}, \\ A_t^I &= e^A X^I, \\ X_I &= \tfrac{1}{3}e^A H_I, \end{aligned} \tag{49}$$

with

$$e^{-\frac{3}{2}A} = \mathcal{V}(Y) = \frac{1}{6}C_{IJK}Y^I Y^J Y^K. \tag{50}$$

In addition to the explicit factor e^{-2igt} in the metric, time dependence also arises in the harmonic functions

$$H_I(t,\vec{x}) = H_I(e^{-igt}\vec{x}) = h_I + e^{2igt}\sum_{j=1}^{N}\frac{q_{I\,j}}{|\vec{x}-\vec{x}_j|^2}. \tag{51}$$

This inclusion of time dependence naturally generalizes the multi-center solution to the ungauged theory, (9) and (10).

This solution may be made more concrete by considering the so-called $STU = 1$ model ($X^1 = S$, $X^2 = T$, $X^3 = U$). From (49), we find

$$e^{-A}TU = H_0, \qquad e^{-A}SU = H_1, \qquad e^{-A}ST = H_2, \tag{52}$$

where H_0, H_1 and H_2 are harmonic functions of the form (51). Eqn. (52) together with the fact that $STU = 1$ implies the following solution for the metric and the moduli fields:

$$e^{-3A} = H_0 H_1 H_2 \tag{53}$$

and

$$S = \left(\frac{H_1 H_2}{H_0^2}\right)^{\frac{1}{3}}, \qquad T = \left(\frac{H_0 H_2}{H_1^2}\right)^{\frac{1}{3}}, \qquad U = \left(\frac{H_0 H_1}{H_2^2}\right)^{\frac{1}{3}}. \tag{54}$$

The gauge fields are then given simply by

$$A_t^1 = \frac{1}{H_0}, \qquad A_t^2 = \frac{1}{H_1}, \qquad A_t^3 = \frac{1}{H_2}. \tag{55}$$

It may be noted that this construction of time-dependent multi-center black hole solutions has a straightforward generalization to arbitrary dimensional AdS black holes. The most straightforward cases would be to four and seven dimensions, where the corresponding gauged supergravity theories have been well studied. As a generalization, one may also imagine seeking multi-centered p-brane solutions asymptotic to AdS. However in this case even single p-branes in AdS have not yet been fully explored (but see [18, 19] for the case of a magnetic string in AdS_5). Recently, a procedure was given in [20, 21] to lift certain multi-centered solutions in Poincaré supergravity to corresponding solutions to gauged supergravities. Starting from a multi-centered $(p-1)$-brane in $(d-1)$ dimensions, this configuration may be lifted to yield a multi p-brane solution in AdS_d, with a resulting metric in "horospheric" form, *i.e.* with a metric factor e^{-2gz} instead of e^{-2igt} where z is the (spatial) lifting coordinate. In the present case, we have little choice but to use time in this construction, as it is the only coordinate longitudinal to a black hole solution. This suggests that the multi-center solutions we have found may be viewed as lifted instantons of a $(d-1)$-dimensional theory. It is because these are instantons that one ends up with an Euclideanized version of the theory. Finally, it remains an open issue to construct true multi-centered solutions of the gauged supergravity theory without resorting to any Wick rotation or use of an imaginary coupling.

Acknowledgments

We wish to thank F. Dowker for clarifying the issue of having the gauge potential become imaginary in the Wick rotated theory. JTL wishes to thank the Center for Advanced Mathematical Sciences (CAMS) at the American University of Beirut, where part of this work was performed. This research was supported in part by DOE Grant DE-FG02-95ER40899 Task G.

References

[1] K. Behrndt, A. H. Chamseddine and W. A. Sabra, *BPS black holes in $N = 2$ five dimensional AdS supergravity*, Phys. Lett. **B442** (1998) 97 [hep-th/9807187].

[2] K. Behrndt, M. Cvetič and W. A. Sabra, *Non-extreme black holes of five dimensional $N = 2$ AdS supergravity*, Nucl. Phys. **B553** (1999) 317 [hep-th/9810227].

[3] D. Kastor and J. Traschen, *Cosmological multi-black hole solutions*, Phys. Rev. **D47** (1993) 5370 [hep-th/9212035].

[4] L. A. J. London, *Arbitrary dimensional cosmological multi-black holes*, Nucl. Phys. **B434** (1995) 709.

[5] G. W. Gibbons, D. Kastor, L. A. J. London, P. K. Townsend and J. Traschen, *Supersymmetric selfgravitating solitons*, Nucl. Phys. **B416** (1994) 850 [hep-th/9310118].

[6] F. R. Tanghelini, Nuovo Cimento **27** (1963) 636.

[7] W. A. Sabra, *General BPS Black Holes In Five-Dimensions*, Mod. Phys. Lett. **A13** (1998) 239 [hep-th/9708103].

[8] A. H. Chamseddine and W. A. Sabra, *Metrics Admitting Killing Spinors in Five Dimensions*, Phys. Lett. **B426** (1998) 36 [hep-th/9801161].

[9] A. H. Chamseddine and W. A. Sabra, *Calabi-Yau black holes and enhancement of supersymmetry in five dimensions*, Phys. Lett. **B460** (1999) 63 [hep-th/9903046].

[10] S. Ferrara and R. Kallosh, *Universality of supersymmetric Attractors*, Phys. Rev. **D54** (1996) 1525 [hep-th/9603090].

[11] W. A. Sabra, *Anti-de Sitter BPS black holes in $N = 2$ gauged supergravity*, Phys. Lett. **B458** (1999) 36 [hep-th/9903143].

[12] M. Günaydin, G. Sierra and P. K. Townsend, *The Geometry Of N=2 Maxwell-Einstein Supergravity And Jordan Algebras*, Nucl. Phys. **B242** (1984) 244.

[13] M. Günaydin, G. Sierra and P. K. Townsend, *Gauging The $D = 5$ Maxwell-Einstein Supergravity Theories: More On Jordan Algebras*, Nucl. Phys. **B253** (1985) 573.

[14] J. Gutowski and G. Papadopoulos, *The dynamics of very special black holes*, Phys. Lett. **B472** (2000) 45 [hep-th/9910022].

[15] M. J. Duff, H. Lü and C. N. Pope, *The black branes of M-theory*, Phys. Lett. **B382** (1996) 73 [hep-th/9604052].

[16] M. J. Duff and J. T. Liu, *Anti-de Sitter black holes in gauged $N = 8$ supergravity*, Nucl. Phys. **B554** (1999) 237 [hep-th/9901149].

[17] L. J. Romans, *Supersymmetric, cold and lukewarm black holes in cosmological Einstein-Maxwell theory*, Nucl. Phys. **B383** (1992) 395 [hep-th/9203018].

[18] A. H. Chamseddine and W. A. Sabra, *Magnetic strings in five dimensional gauged supergravity theories*, Phys. Lett. **B477** (2000) 329 [hep-th/9911195].

[19] D. Klemm and W. A. Sabra, *Supersymmetry of black strings in $D = 5$ gauged supergravities*, Phys. Rev. **D62** (2000) 024003 [hep-th/0001131].

[20] H. Lü and C. N. Pope, *Branes on the brane*, hep-th/0008050.

[21] M. Cvetič, H. Lü and C. N. Pope, *Brane-world Kaluza-Klein reductions and branes on the brane*, hep-th/0009183.